10 — Radiation & Matter [*Revision* : 1.3]

- Specific intensity is constant along ray through empty space
- However, matter **interacts** with radiation to change I_λ
- Three principal processes:
  - **Absorption**: photon is destroyed, energy goes into excitation/kinetic energy of matter
  - **Emission**: photon is created, energy comes from excitation/kinetic energy of matter
  - **Scattering**: photon's direction, and possibly wavelength, is changed by 'collision' with matter
- Cross section
  - Consider photon beam of cross-section $\mathrm{d}A$, impingent on black (completely-absorbing) object with cross-section σ. Fraction of photons absorbed is $\sigma/\mathrm{d}A$
  - Probability that any individual photon absorbed is

$$P = \frac{\sigma}{\mathrm{d}A}$$

  - Apply similar reasoning to atomic-scale absorbers & scatterers: probabilty that individual photon interacts (absorb or scatter) is same, but σ is now the **interaction cross section** (units of area)
- Mean free path
  - Consider photons travelling through slab with face area $\mathrm{d}A$ and (infinitessimal) thickness $\mathrm{d}s$, containing N particles
  - Probability that individual photon interacts is

$$P = N\frac{\sigma}{\mathrm{d}A} = n\sigma\mathrm{d}s$$

    where $n \equiv N/(\mathrm{d}A\mathrm{d}s)$ is number of particles per unit volume
  - Important result: probability of interaction per unit length is $P/\mathrm{d}s = n\sigma$.
  - If beam contains N_p photons when it enters the slab, then on exiting PN_p will have interacted; number remaining in beam is

$$N_\mathrm{p}(s + \mathrm{d}s) = N_\mathrm{p}(s) - PN_\mathrm{p}(s) = N_\mathrm{p}(s)(1 - P)$$

    where s is location where photons enter slab
  - Rearranging:

$$N_\mathrm{p}(s + \mathrm{d}s) - N_\mathrm{p}(s) = -N_\mathrm{p}(s)P = -N_\mathrm{p}(s)n\sigma\mathrm{d}s$$

  - In limit $\mathrm{d}s \to 0$:

$$\frac{N_\mathrm{p}(s + \mathrm{d}s) - N_\mathrm{p}(s)}{\mathrm{d}s} = \frac{\mathrm{d}N_\mathrm{p}}{\mathrm{d}s} = -N_\mathrm{p}n\sigma$$

    (**photon transport equation**)
  - Solving:

$$N_\mathrm{p}(s) = N_{\mathrm{p},0}\mathrm{e}^{-n\sigma s}$$

    where $N_{\mathrm{p},0}$ is constant of integration

– Corrollary: probability that photon travels macroscopic distance s without interaction:

$$Q(s) = \frac{N_{\rm p}(s)}{N_{\rm p,0}} = {\rm e}^{-n\sigma s}$$

– Also, probabilty that an interaction takes place in the interval $(s, s + {\rm d}s)$ is probability photon travels distance s *without* interaction (see above), times probability it interacts in subsequent ${\rm d}s$:

$$P(s){\rm d}s = Q(s)n\sigma{\rm d}s = n\sigma{\rm e}^{-n\sigma s}{\rm d}s$$

– Most probable distance before interaction:

$$\langle s\rangle = \int_0^\infty n\sigma s{\rm e}^{-n\sigma s}{\rm d}s = \frac{1}{n\sigma};$$

This is the **mean free path** of photons

- Opacity
  - For medium of density ρ, number density n is

$$n = \frac{\rho}{\mu}$$

where μ is **mean molecular weight**

  - For absorption processes, solution of transport equation above can also be written

$$N_{\rm p}(s) = N_{\rm p,0}{\rm e}^{-\kappa\rho s}$$

where κ is the **opacity**.

  - Same for scattering proccesses, but different symbol typically used for opacity (often $\tilde{\sigma}$, which is confusing!)
  - Can also write

$$N_{\rm p}(s) = N_{\rm p,0}{\rm e}^{-\tau(s)}$$

where $\tau \equiv \kappa\rho s$ is **optical thickness**.

  - **Optically thin**: $\tau \ll 1$:

$$N_{\rm p}(s) \approx N_{\rm p,0}(1 - \tau)$$

(varies linearly with τ)

  - **Optically thick**: $\tau \gg 1$
  - Optical depth $\tau = 1$ equivalent to one mean free path:

$$\kappa\rho s = 1 \longleftrightarrow s = \frac{1}{\kappa\rho} = \langle s\rangle$$

  - So far, analysis for uniform (constant ρ, κ) slab. For non-uniform medium, define

$${\rm d}\tau = \kappa(s)\rho(s){\rm d}\tau$$

so that

$$\tau = \int \kappa(s)\rho(s){\rm d}s$$

Same formulae then apply, e.g.

$$N_{\rm p}(s) = N_{\rm p,0}{\rm e}^{-\tau(s)}$$

- In language of specific intensity, above equations become

$$I(s) = I_0 \mathrm{e}^{-\tau(s)}$$

- Since κ generally depends on wavelength, more-general form is

$$I_\lambda(s) = I_{\lambda,0} \mathrm{e}^{-\tau_\lambda(s)}$$

where

$$\tau = \int \kappa(s)\rho(s)\mathrm{d}s$$

and κ_λ is **monochromatic opacity**

- Emissivity
  - Consider radiation travelling through same slab with (infinitessimal) thickness $\mathrm{d}s$
  - Change in specific intensity traveling through slab is

$$\mathrm{d}I_\lambda = j_\lambda \mathrm{d}s$$

where j_λ is **emissivity**: amount of radiation emitted per second, per unit wavelength interval, per unit volume, per unit solid angle, in certain direction.

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